- If $x(n) \overset{\text{ZT}}{\longleftrightarrow} X(z)$ with ROC R, then

$$x^*(n) \overset{\text{ZT}}{\longleftrightarrow} X^*(z^*) \quad \text{with ROC } R.$$

- This is known as the conjugation property of the z transform.

- If $x_1(n) \overset{\text{ZT}}{\longleftrightarrow} X_1(z)$ with ROC R_1 and $x_2(n) \overset{\text{ZT}}{\longleftrightarrow} X_2(z)$ with ROC R_2, then

$$x_1 * x_2(n) \overset{\text{ZT}}{\longleftrightarrow} X_1(z)X_2(z) \quad \text{with ROC containing } R_1 \cap R_2.$$

- This is known that the convolution (or time-domain convolution) property of the z transform.
- The ROC always contains the intersection but can be larger than the intersection (if pole-zero cancellation occurs.(
- Convolution in the time domain becomes *multiplication* in the z domain.
- This can make dealing with LTI systems much easier in the z domain than in the time domain.

- If $x(n) \overset{\text{ZT}}{\longleftrightarrow} X(z)$ with ROC R, then

$$nx(n) \overset{\text{ZT}}{\longleftrightarrow} -z\frac{d}{dz}X(z) \quad \text{with ROC } R.$$

- This is known as the z-domain differentiation property of the z transform.

- If $x(n) \overset{\text{ZT}}{\longleftrightarrow} X(z)$ with ROC R, then

$$x(n) - x(n-1) \overset{\text{ZT}}{\longleftrightarrow} (1 - z^{-1})X(z) \quad \text{for ROC containing } R \cap |z| > .0$$

- This is known as the differencing property of the z transform. Differencing
- in the time domain becomes multiplication by $1 - z^{-1}$ in the z domain.

  This can make dealing with difference equations much easier in the z
- domain than in the time domain.

- If $x(n) \stackrel{\text{ZT}}{\longleftrightarrow} X(z)$ with ROC R, then

$$\sum_{k=-\infty}^{n} x(k) \stackrel{\text{ZT}}{\longleftrightarrow} \frac{z}{z-1} X(z) \text{ for ROC containing } R \cap |z| > 1.$$

- This is known as the accumulation property of the z transform.

- For a sequence x with z transform X, if x is causal, then

$$x(0) = \lim_{z \to \infty} X(z).$$

- This result is known as the initial-value theorem.

- For a sequence x with z transform X, if x is causal and $\lim_{n\to\infty} x(n)$ exists, then

$$\lim_{n\to\infty} x(n) = \lim_{z\to 1}[(z-1)X(z)]$$

- This result is known as the final-value theorem.

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Section 11.4

Determination of Inverse Z Transform

- Recall that the inverse z transform x of X is given by

$$x(n) = \frac{1}{2\pi j} \oint_{\Gamma} X(z) z^{n-1} dz,$$

where Γ is a counterclockwise closed circular contour centered at the origin and with radius r such that Γ is in the ROC of X.

- Unfortunately, the above contour integration can often be *quite tedious* to compute.
- Consequently, we do not usually compute the inverse z transform directly using the above equation.
- For rational functions, the inverse z transform can be more easily computed using *partial fraction expansions*.
- Using a partial fraction expansion, we can express a rational function as a sum of lower-order rational functions whose inverse z transforms can typically be found in tables.

Section 11.5

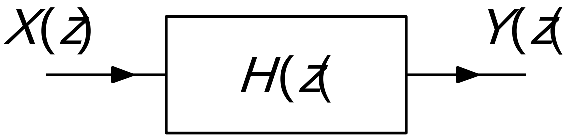
Z Transform and LTI Systems

- Consider a LTI system with input x, output y, and impulse response h, and let X, Y, and H denote the z transforms of x, y, and h, respectively.
- Since $y(n) = x * h(n)$, the system is characterized in the z domain by

$$Y(z) = X(z)H(z).$$

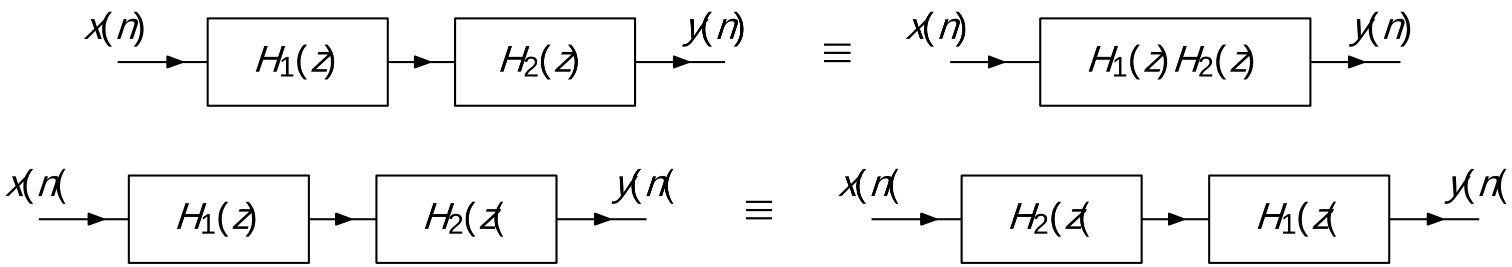
- As a matter of terminology, we refer to H as the system function (or transfer function) of the system (i.e., the system function is the z transform of the impulse response).
- When viewed in the z domain, a LTI system forms its output by multiplying its input with its system function.
- A LTI system is completely characterized by its system function H.
- If the ROC of H includes the unit circle $|z| = 1$, then $H(z)|_{z=e^{j\Omega}}$ is the *frequency response* of the LTI system.

- Consider a LTI system with input x, output y, and impulse response h, and let X, Y, and H denote the z transforms of x, y, and h, respectively.
- Often, it is convenient to represent such a system in block diagram form in the z domain as shown below.

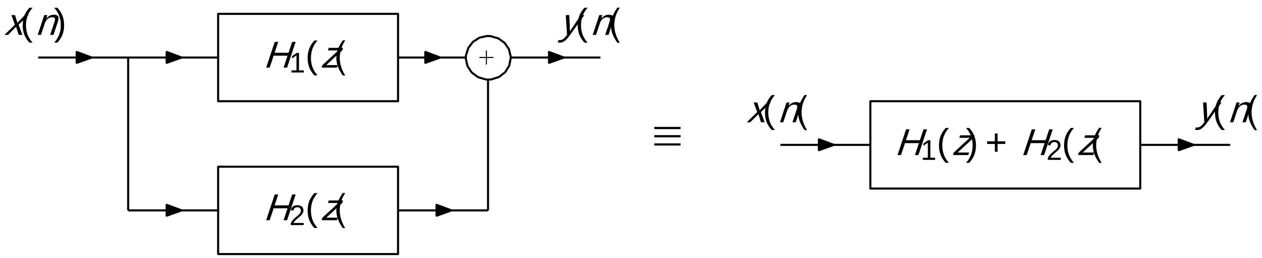


- Since a LTI system is completely characterized by its system function, we typically label the system with this quantity.

- The *series* interconnection of the LTI systems with system functions H_1 and H_2 is the LTI system with system function $H = H_1H_2$. That is, we have the equivalences shown below.



- The *parallel* interconnection of the LTI systems with impulse responses H_1 and H_2 is a LTI system with the system function $H = H_1 + H_2$. That is, we have the equivalence shown below.



- If a LTI system is *causal*, its impulse response is causal, and therefore *right sided*. From this, we have the result below.
- **Theorem.** A LTI system is *causal* if and only if the ROC of the system function is the *exterior of a circle including infinity*.
- **Theorem.** A LTI system with a *rational* system function H is causal if and only if
  1. the ROC is the exterior of a circle *outside the outermost pole*; and with
  2. $H(z)$ expressed as a ratio of polynomials in z the order of the numerator polynomial *does not exceed* the order of the denominator polynomial.